BACKGROUND ON MEASUREMENTS AND CALCULATIONS

01 **Introduction**

This section provides a background in the mathematical rules and procedures used in making measurements and performing calculations. Topics include:

- Units: Metric vs. English
- Mass vs. Weight
- Balances and Scales
- Rounding
- Significant Figures
- Accuracy and Precision
- Tolerance

Also included is discussion of real-world applications in which the mathematical rules and procedures may not be followed.

02 **Units: Metric vs. English**

Basic units in SI include:

Length: meter, m
Mass: kilogram, kg
Time: second, s

Derived units in SI include:

Force: Newton, N

SI units

03 The bulk of this document uses dual units. Metric units are followed by Imperial, more commonly known as English, units in parentheses. For example: 25 mm (1 in.). Exams are presented in metric or English.

04 Depending on the situation, some conversions are exact, and some are approximate. One inch is exactly 25.4 mm. If a procedure calls for measuring to the closest 1/4 in., however, 5 mm is close enough. We do not have to say 6.35 mm. That is because 1/4 in. is half way between 1/8 in. and 3/8 in. – or half way between 3.2 and 9.5 mm. Additionally, the tape measure or rule used may have 5 mm marks, but may not have 1 mm marks and certainly will not be graduated in 6 mm increments.

Metric	English
25 mm	1 in.
1 kg	2.2 lb
1000 kg/m^3	62.4 lb/ft^3
25 MPa	3600 lb/in.^{2}

Some approximate conversions

In SI (Le Systeme International d'Unites), the basic unit of mass is the kilogram (kg) and the basic unit of force, which includes weight, is the Newton (N). Mass in this document is given in grams (g) or kg. See the section below on "Mass vs. Weight" for further discussion of this topic.

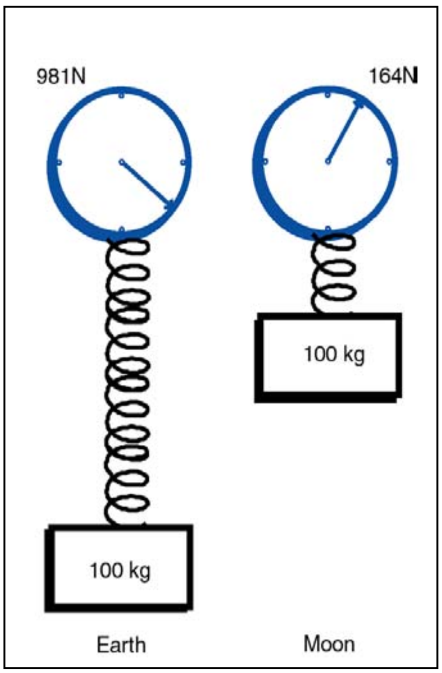
Mass vs. Weight

05 The terms mass, force, and weight are often confused. Mass, m, is a measure of an object's material makeup, and has no direction. Force, F, is a measure of a push or pull, and has the direction of the push or pull. Force is equal to mass times acceleration, a.

06

$$F = ma$$

Weight, W, is a special kind of force, caused by gravitational acceleration. It is the force required to suspend or lift a mass against gravity. Weight is equal to mass times the acceleration due to gravity, g, and is directed toward the center of the earth.



Comparison of mass and weight

$$W = mg$$

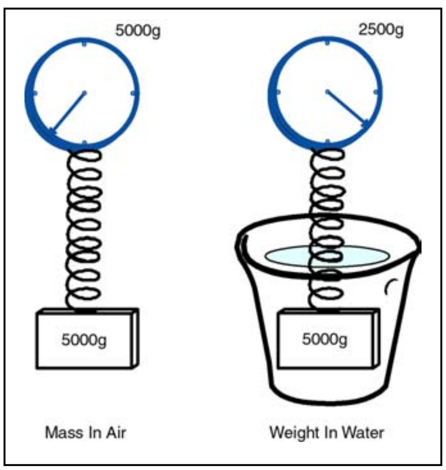
In SI, the basic unit of mass is the kilogram (kg), the units of acceleration are meters per square second (m/s^2), and the unit of force is the Newton (N). Thus a person having a mass of 84 kg subject to the standard acceleration due to gravity, on earth, of 9.81 m/s^2 would have a weight of:

$$W = (84.0\ \text{kg})(9.81\ \text{m/s}^2) = 824\ \text{kg-m/s}^2 = 824\ \text{N}$$

07 In the English system, mass can be measured in pounds-mass (lb_m), while acceleration is in feet per
08 square second (ft/s^2), and force is in pounds-force (lb_f). A person weighing 185 lb_f on a scale has a mass of 185 lb_m when subjected to the earth's standard gravitational pull. If this person were to go to the moon, where the acceleration due to gravity is about one-sixth of what it is on earth, the person's weight would be about 31 lb_f, while his or her mass would remain 185 lb_m. Mass does not depend on location, but weight does.

While the acceleration due to gravity does vary with position on the earth (latitude and elevation), the variation is not significant except for extremely precise work – the manufacture of electronic memory chips, for example.

As discussed above, there are two kinds of pounds, lb_m and lb_f. In laboratory measurements of mass, the gram or kilogram is the unit of choice. But, is
09 this mass or force? Technically, it depends on the instrument used, but practically speaking, mass is the result of the measurement. When using a scale, force is being measured – either electronically by the stretching of strain gauges or mechanically by the stretching of a spring or other device. When using a balance, mass is being measured, because the mass of the object is being compared to a known mass built into the balance.



Submerged weight

In this document, mass, not weight, is used in test procedures except when determining "weight" in water. When an object is submerged in water (as is done in specific gravity tests), the term weight is used. Technically, what is being measured is the force the object exerts on the balance or scale while the object is submerged in water (or the submerged weight). This force is actually the weight of the object less the weight of the volume of water displaced.

In summary, whenever the common terms "weight"
10 and "weighing" are used, the more appropriate
11 terms "mass" and "determining mass" are usually implied, except in the case of weighing an object submerged in water.

12

Balances and Scales

Balances, technically used for mass determinations, and scales, used to weigh items, were discussed briefly above in the section on "Mass vs. Weight." In field operating procedures, we usually do not differentiate between the two types of instruments. When using either one for a material or object in air, we are determining mass. For those procedures in which the material or object is suspended in water, we are determining weight in water.

13 AASHTO recognizes two general categories of instruments. Standard analytical balances are used in laboratories. For most field operations, general purpose balances and scales are specified. Specifications for both categories are shown in
14 Tables 1 and 2.

Table 1
Standard Analytical Balances

Class	Capacity	Readability and Sensitivity	Accuracy
A	200 g	0.0001 g	0.002 g
B	200 g	0.001 g	0.002 g
C	1200 g	0.01 g	0.02 g

Table 2
General Purpose Balances and Scales

Class	Principal Sample Mass	Readability and Sensitivity	Accuracy
G2	2 kg or less	0.1 g	0.1 g or 0.1 percent
G5	2 kg to 5 kg	1 g	1 g or 0.1 percent
G20	5 kg to 20 kg	5 g	5 g or 0.1 percent
G100	Over 20 kg	20 g	20 g or 0.1 percent

15 Rounding

Numbers are commonly rounded up or down after measurement or calculation. For example, 53.67 would be rounded to 53.7 and 53.43 would be rounded to 53.4, if rounding were required. The first number was rounded up because 53.67 is closer to 53.7 than to 53.6. Likewise, the second number was rounded down because 53.43 is closer to 53.4 than to 53.5. The reasons for rounding are covered in the next section on "Significant Figures."

If the number being rounded ends with a 5, two possibilities exist. In the more mathematically sound approach, numbers are rounded up or down depending on whether the number to the left of the 5 is odd or even. Thus, 102.25 would be rounded down to 102.2, while 102.35 would be rounded up to 102.4. This procedure avoids the bias that would exist if all numbers ending in 5 were rounded up or all numbers were rounded down. In some calculators, however, all rounding is up. This does result in some bias, or skewing of data, but the significance of the bias may or may not be significant to the calculations at hand.

Significant Figures

- General

16 A general purpose balance or scale, classified as G20 in AASHTO M 231, has a capacity of 20,000 g and an accuracy requirement of ±5 g. A mass of 18,285 g determined with such an instrument could actually range from 18,280 g to 18,290 g. Only <u>four</u> places in the measurement are significant. The fifth (last) place is <u>not</u> significant since it may change.

17 Mathematical rules exist for handling significant figures in different situations.
An example in Metric(**m**) or English(**ft**), when performing addition and subtraction, the number of significant figures in the sum or difference is determined by the least precise input. Consider the three situations shown below:

<u>Situation 1</u>	<u>Situation 2</u>	<u>Situation 3</u>
35.67	143.903	162
<u>+ 423.938</u>	<u>- 23.6</u>	+33.546
		<u>- .022</u>
= 459.61	= 120.3	= 196
not 459.608	not 120.303	not 195.524

Rules also exist for multiplication and division. These rules, and the rules for mixed operations involving addition, subtraction, multiplication, and/or division, are beyond the scope of these materials. AASHTO covers this topic to a certain extent in the section called “Precision” or “Precision and Bias” included in many test methods, and the reader is directed to those sections if more detail is desired.

18 • Real World Limitations

While the mathematical rules of significant digits have been established, they are not always followed. For example, AASHTO Method of Test T 176, *Plastic Fines in Graded Aggregates and Soils by the Use of the Sand Equivalent Test*, prescribes a method for rounding and significant digits in conflict with the mathematical rules.

In this procedure, readings and calculated values are always rounded up. A clay reading of 7.94 would be rounded to 8.0 and a sand reading of 3.21 would be rounded to 3.3. The rounded numbers are then used to calculate the Sand Equivalent, which is the ratio of the two numbers multiplied by 100. In this case:

$$\frac{3.3}{8.0} \times 100 = 41.250...,$$

rounded to 41.3 and reported as 42

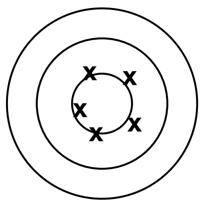
$$(\text{Not}: \frac{3.21}{7.94} \times 100 = 40.428...,$$

rounded to 40.0 and reported as 40)

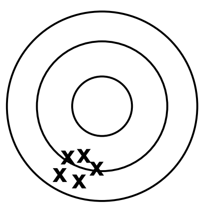
It is extremely important that engineers and technicians understand the rules of rounding and significant digits just as well as they know procedures called for in standard test methods.

Accuracy and Precision

19 Although often used interchangeably, the terms
accuracy and precision do not mean the same thing.
In an engineering sense, accuracy denotes nearness
to the truth or some value accepted as the truth,
20 while precision relates to the degree of refinement
or repeatability of a measurement.



ACCURATE BUT NOT PRECISE, SCATTERED



PRECISE BUT NOT ACCURATE, BIASED

Two bullseye targets are shown to the left. The upper one indicates hits that are scattered and, yet, are very close to the center. The lower one has a tight pattern, but all the shots are biased from the center. The upper one is more accurate, while the lower one is more precise. A biased, but precise, instrument can often be adjusted physically or mathematically to provide reliable single measurements. A scattered, but accurate, instrument can be used if enough measurements are made to provide a valid average.

Consider the measurement of the temperature of
boiling water at standard atmospheric pressure by
21 two thermometers. Five readings were taken with
each, and the values were averaged.

Thermometer No. 1	Thermometer No. 2
101.2° 214.2°	100.6° 213.1°
101.1° 214.0°	99.2° 210.6°
101.2° 214.2°	98.9° 210.0°
101.1° 214.0°	101.0° 213.8°
101.2° 214.2°	100.3° 212.5°
AVG = 101.2° 214.2°	AVG = 100.0° 212°

No. 1 shows very little fluctuation, but is off the
22 known boiling point (100°C or 212°F) by 1.2°C or
2.2°F. No. 2 has an average value equal to the
known boiling point, but shows quite a bit of
fluctuation. While it might be preferable to use
neither thermometer, thermometer No. 1 could be

employed if 1.2°C or 2.2°F were subtracted from
each measurement. Thermometer No. 2 could be
23 used if enough measurements were made to provide
a valid average.

24 Engineering and scientific instruments should be
calibrated and compared against reference standards
periodically to assure that measurements are
accurate. If such checks are not performed, the
accuracy is uncertain, no matter what the precision.
Calibration of an instrument removes fixed error,
25 leaving only random error for concern.

Tolerance

Dimensions of constructed or manufactured objects,
26 including laboratory test equipment, cannot be
specified exactly. Some tolerance must be allowed.
Thus, procedures for including tolerance in
addition/subtraction and multiplication/division
operations must be understood.

- Addition and Subtraction

27

  When adding or subtracting two numbers that individually have a tolerance, the tolerance of the sum or difference is equal to the sum of the individual tolerances.

  An example in Metric(**m**) or English(**ft**), if the distance between two points is made up of two parts, one being 113.361 ±0.006 and the other being 87.242 ±0.005 then the tolerance of the sum (or the difference) is:

  $$(0.006) + (0.005) = 0.011$$

  and the sum would be 200.603 ±0.011.

- Multiplication and Division

28 To demonstrate the determination of tolerance
again in either Metric(**m**) or English(**ft**) for the
product of two numbers, consider determining
the area of a rectangle having sides of 76.254

±0.009 and 34.972 ±0.007. The percentage variations of the two dimensions are:

$$\frac{0.009}{76.254} \times 100 = 0.01\% \qquad \frac{0.007}{34.972} \times 100 = 0.02\%$$

The sum of the percentage variations is 0.03 percent – the variation that is employed in the area of the rectangle:

Area =
2666.8 (m^2 or ft^2) ±0.03 percent = 2666.8 ± 0.8 (m^2 or ft^2).

- Real World Applications

29 Tolerances are used whenever a product is manufactured. For example, the mold used for determining soil density in AASHTO T 99 has a diameter of 101.60 ±0.41 mm(4.000 ±0.016 in) and a height of 116.43 ±0.13 mm(4.584 ±0.005 in).

Using the smaller of each dimension results in a volume of:

(π/4) $(101.19\text{ mm})^2$ (116.30 mm) =
935,287 mm^3 or 0.000935 m^3

(π/4) $(3.984\text{ in})^2$ (4.579 in) =
57.082in^3 or 0.0330 ft^3

Using the larger of each dimension results in a volume of:

(π/4) $(102.01\text{ mm})^2$ (116.56 mm) =
952,631 mm^3 or 0.000953 m^3

(π/4) $(4.016\text{ in})^2$ (4.589 in) =
58.130 in^3 or 0.0336 ft^3

The average value is 0.000944 m^3 (0.0333), and AASHTO T 99 specifies a volume of:

0.000943 ±0.000008 m^3
or a range of
0.000935 to 0.000951 m^3

0.0333 ±0.0003 ft^3
or a range of
0.0330 to 0.0336 ft^3

Because of the variation that can occur, some agencies periodically calibrate molds, and make adjustments to calculated density based on those calculations.

Summary

Mathematics has certain rules and procedures for making measurements and performing calculations that are well established. So are standardized test procedures. Sometimes these agree, but occasionally, they do not. Engineers and technicians must be familiar with both, but must follow test procedures in order to obtain valid, comparable results.